PREDICTION THE COMPETITIVE STRENGTH INDEX OF BRAND PRODUCT ON BASE OF FUZZY LOGIC

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1 Introduction

The competitive strength index is usually predicted via the weighted sum of particular technical and economical figures of the product. That approach does not allow the processing of expert knowledge and uncertain source information like: “Bad image”, “Reasonable price” etc. Experienced brand managers are often forced to make decisions on the basis of following expert judgments: “if the product price is low, and the product quality is high, and the brand image is attractive, then the competitive strength is very high”. Fuzzy set theory [1, 2] provides an easy way of transforming such expert knowledge to a mathematical model. This paper presents a fuzzy model for prediction of the competitive strength index of brand products.

2 Problem Statement

Let us define the competitive strength index (Q) of a brand product as a number from [0, 100]. This number shows an ability of the brand product to hold the competition among other ones on the given market. Hence, the competitive strength index is proportional to a market segment of the product. Hierarchical interconnection between influence factors (x) and competitive strength index (Q) is proposed as follows:

\[ Q = f_0(x_1, y_1, y_2, y_3), \]
\[ y_1 = f_1(x_2, x_3, x_4), \]
\[ y_2 = f_2(x_5, x_6, x_7), \]
\[ y_3 = f_3(x_8, x_9, x_{10}), \]

where \( f(\cdot) \) denotes an input-output mapping in a fuzzy knowledge base form; \( y_1 \) is the quality of the brand product; \( y_2 \) is the image of the brand product; \( y_3 \) is the service connected with the brand product; \( x_1 \) is the price of the brand product; \( x_2 \) is the quality of the product project; \( x_3 \) is the quality of the productive technologies; \( x_4 \) is the level of the people ware; \( x_5 \) is the enterprise status; \( x_6 \) is level of advertising campaign; \( x_7 \) is level of the reclamations; \( x_8 \) is the level of the selling service; \( x_9 \) is the level of the after-sale service; \( x_{10} \) is the bonus attached to the brand-product.

The value of a particular factor \( x_i \) (\( i = 1, 10 \)) will be determined as a deviation (in percentages) from average figures of rival brand products on the given market.

3 Fuzzy Knowledge Bases

Mamdani-type fuzzy knowledge bases represent the “input-output” ties in (2), (3), and (4). Tables 1–3 show fragments of those bases. There are 49 fuzzy rules in total. The rules are produced a market expert.

Relation (1) is modeled by the following Sugeno-type fuzzy rules:

\[ Q = -0.08x_1 + 0.03y_1 + 0.025y_2 + 0.055y_3 + 14; \]
\[ Q = -0.35x_1 + 0.4y_1 + 0.28y_2 + 0.05y_3 + 50; \]
\[ Q = -0.06x_1 + 0.06y_1 + 0.06y_2 + 0.08y_3 + 80. \]
Each rule in (5) corresponds to one sale strategy. For the first strategy, the values of the price, and the quality, and the image, and the service are bad, regarding a customer’s point of view. They are average for the second type of sale strategy, and they are good in case of the third strategy. It is assumed, that an elasticity of the competitive strength index on factor changing is constant inside single sale strategy. Coefficients in rule consequents in (5) correspond to competitive strength sensitivity for relevant factors. They are calculated via Saaty’s method [4] on base of paired comparisons the importance of the factors.

Table 1. Fuzzy expert knowledge about relation (2)

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<tr>
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Table 2. Fuzzy expert knowledge about relation (3)

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Table 3. Fuzzy expert knowledge about relation (4)

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Figure 1 shows membership functions of terms from the fuzzy knowledge bases. The following Gaussian curve membership function is used:

$$
\mu^I(x) = \exp\left(-\frac{(x-z)^2}{2c^2}\right),
$$

(6)

where $\mu^I(x)$ is a membership function of variable $x$ to term $t$; $z$ is the coordinate of the curve center; $c$ is the coefficient of the curve concentration.
4 Fuzzy Inference

Typical Mamdani-type and Sugeno-type fuzzy inference algorithms are used [2, 3]. They are specified below.

Let us describe a Mamdani-type fuzzy rule base for relation \( y = f(x_1, x_2, \ldots, x_n) \) as follows:

\[
\text{If } x_1 = \tilde{a}_1^j \text{ and } x_2 = \tilde{a}_2^j \text{ and } \ldots \text{ and } x_n = \tilde{a}_n^j, \text{ then } y = d_j, \quad j=1, m, \tag{7}
\]

where \( \tilde{a}_i^j \) and \( d_j \) are fuzzy terms with membership functions \( \mu_i^j(x_i) \) and \( \mu^j(y) \), \( x_i \in [x_i^-, x_i^+] \), \( y \in [y_-, y^+] \), \( i=1, n \).

The following fuzzy value of output \( y \) corresponds to the given input vector \( X = (x_1, x_2, \ldots, x_n) \):

\[
\tilde{y} = \left( \frac{\mu_1^j(X)}{d_1}, \frac{\mu_2^j(X)}{d_2}, \ldots, \frac{\mu_m^j(X)}{d_m} \right),
\]

where \( \mu_j^j(X) = \mu_i^j(x_1) \wedge \mu_i^j(x_2) \wedge \ldots \wedge \mu_i^j(x_n) \) is a membership grade of vector \( X \) to fuzzy set \( \tilde{d}_j \), according to the \( j \)-th rule of knowledge base (7); \( \wedge \) denotes t-norm (multiplication in this paper).

The membership function of fuzzy number \( \tilde{y} \) on support \([y_-, y^+]\) is calculated as follows:

\[
\mu_{\tilde{y}}(y) = \max_{j=1, m} \min \left( \mu_1^j(X), \mu_2^j(y) \right), \quad j=1, m.
\]

The crisp value of \( \tilde{y} \) is computed by the following defuzzification:

\[
y = \int y \mu_{\tilde{y}}(y) dy \bigg/ \int \mu_{\tilde{y}}(y) dy.
\]

Let us describe the Sugeno-type fuzzy rule base for relation \( y = f(x_1, x_2, \ldots, x_n) \) as follows:

\[
\text{If } x_1 = \tilde{a}_1^j \text{ and } x_2 = \tilde{a}_2^j \text{ and } \ldots \text{ and } x_n = \tilde{a}_n^j, \text{ then } y = d_j, \quad j=1, m, \tag{8}
\]

where \( d_j = b_0^j + b_1^j \cdot x_1 + b_2^j \cdot x_2 + \ldots + b_p^j \cdot x_n \) is a linear function; \( b_p^j \) is a real number, \( p=0, n \).

The following output fuzzy value corresponds to \( X = (x_1, x_2, \ldots, x_n) \) :

\[
\tilde{y} = \left( \frac{\mu_1^j(X)}{d_1}, \frac{\mu_2^j(X)}{d_2}, \ldots, \frac{\mu_m^j(X)}{d_m} \right),
\]

where \( \mu_j^j(X) = \mu_i^j(x_1) \wedge \mu_i^j(x_2) \wedge \ldots \wedge \mu_i^j(x_n) \) is a membership grade of vector \( X \) to fuzzy set \( \tilde{d}_j \), according to the \( j \)-th rule of knowledge base (7); \( \wedge \) denotes t-norm (multiplication in this paper).
\[
\bar{y} = \left( \frac{\mu_{d_1}(X), \mu_{d_2}(X), \ldots, \mu_{d_m}(X)}{d_1, d_2, \ldots, d_m} \right). 
\]

(9)

The crisp value of (9) is calculated by the following defuzzification:

\[
y = \frac{\sum_{j=1}^{m} \mu_{d_j}(X) \cdot d_j}{\sum_{j=1}^{m} \mu_{d_j}(X)}. 
\]

(10)

5 Examples

Task 1. Experts assessed the figures of brand product “Vodka Podilya” (TM “Sotka”) on Vinnitsa regional market as follows: \(x_1 = 10\%\), \(x_2 = \text{High}\), \(x_3 = \text{Average}\), \(x_4 = \text{Average}\), \(x_5 = \text{Average}\), \(x_6 = -50\%\), \(x_7 = -40\%\), \(x_8 = -30\%\), \(x_9 = \text{Average}\), and \(x_{10} = -80\%\) (data on September, 2004).

As a result of Mamdani fuzzy inferences we obtain the following values of the enlarged influence factors: \(y_1 = 9.15\%\), \(y_2 = 11.9\%\), and \(y_3 = 34.7\%\). Figure 2 illustrates those fuzzy inferences. After applying formula (9) and (10) to fuzzy knowledge base (5), the competitive strength index of “Vodka Podilya” is inferred as \(Q_p = 51.75\).

![Figure 2. Mamdani-type fuzzy logic inferences for task 1](image)

Task 2. Experts assessed the figures of brand product “Vodka Nemiroff Original” (TM “Nemiroff”) on the same market as follows: \(x_1 = 40\%\), \(x_2 = \text{High}\), \(x_3 = 25\%\), \(x_4 = \text{High}\), \(x_5 = \text{High}\), \(x_6 = 70\%\), \(x_7 = -20\%\), \(x_8 = 80\%\), \(x_9 = \text{Average}\), and \(x_{10} = -50\%\) (data on September, 2004).

As a result of the fuzzy modeling we obtain the following value of the competitive strength index of “Vodka Nemiroff Original”: \(Q_N = 62.41\).

Task 3. It is necessary to increase the competitive strength index of “Vodka Podilya” up to \(Q^* \geq 63\) that is higher than the rival level \(Q_N = 62.41\). How to do it with minimal recourses? A manager may change 2 factors in the following frames: \(x_1 \in [-20, 20]\) and \(x_6 \in [-55, 10]\). Increments of factors \(x_1\) and \(x_2\) on 1 request \(c_1 = -10000\text{UAH}\) and \(c_6 = 1500\text{UAH}\).

Figure 3 shows the graphical solution of this optimizing task. 5 dotted lines indicate traces of the goal function with values \(C = -70000, 0, 100000, 168000,\) and \(300000\text{UAH}\). The pentagon points to the optima.

Task 4. How to increase the competitive strength index of “Vodka Podilya” maximally without additional fund? A manager may act as pointed in task 3.

Figure 4 shows the graphical solution of optimizing task 4. The pentagon points to the optima.
6 Identification of the Fuzzy Model

The fuzzy model predicts the following values of the competitive strength index: $Q_P = 51.75$ for “Vodka Podilya” and $Q_N = 62.41$ for “Vodka Nemiroff Original”. Hence, a ratio of their predicted market segments $\beta_P$ and $\beta_N$ is $\beta_N : \beta_P = Q_N : Q_P = 62.41 : 51.75 = 1.21:1$. Cashbooks of two Vinnitsa supermarkets “Da! Market” indicate this ratio as 1.29:1 and 1.41:1. There is a difference between actual and inferred results.

An improvement of the fuzzy model may be done by training the membership functions and the consequents of fuzzy rules. Let’s define the training set as follows:

$$\{(X_{ij}, \beta_{ij}) : i = 1, H, j = 1, M_i\},$$

where $H$ denotes the number of the regional markets; $M_i$ denotes the number of rival brand products on the $i$-th market; $\beta_{ij}$ denotes the actual segment of the $i$-th market for the $j$-th brand product; $X_{ij}$ denotes the vector on the influence factors for the $j$-th brand product on $i$-th market.

According to the theory of fuzzy identification [3, 5] the training is searching the vector of model parameters ($P$) that minimize the difference between actual and predicted market segmentation:

$$\frac{1}{S} \sum_{i=1,H} \sum_{j=1,M_i} (\beta_{ij} - \beta_F(X_{ij}, P))^2 \rightarrow \min,$$

where $\beta_F(X_{ij}, P)$ denotes the predicted market segment for product $X_{ij}$; $S = \sum_{i=1,H} M_i$ is a length of (11).

7 Conclusion

A fuzzy model of brand competitiveness index is proposed. The fuzzy model is based on 4 expert knowledge bases. The model can be the basis for solving various tasks of optimal management of brand products. An example of achievement the desired level of competitive strength with minimal expenses is discussed.

References