PREDICTION THE COMPETITIVE STRENGTH INDEX OF BRAND PRODUCT ON BASE OF FUZZY LOGIC

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1 Introduction

The competitive strength index is usually predicted via the weighted sum of particular technical and economical figures of the product. That approach does not allow the processing of expert knowledge and uncertain source information like: "Bad image", "Reasonable price" etc. Experienced brand managers are often forced to make decisions on the basis of following expert judgments: *"if the product price is low, and the product quality is high, and the brand image is attractive, then the competitive strength is very high"*. Fuzzy set theory [1, 2] provides an easy way of transforming such expert knowledge to a mathematical model. This paper presents a fuzzy model for prediction of the competitive strength index of brand products.

2 Problem Statement

Let us define the competitive strength index (Q) of a brand product as a number from [0, 100]. This number shows an ability of the brand product to hold the competition among other ones on the given market. Hence, the competitive strength index is proportional to a market segment of the product. Hierarchical interconnection between influence factors ($x_1,...,x_{10}$) and competitive strength index (Q) is proposed as follows:

$$Q=f_Q(x_1,y_1,y_2,y_3), \tag{1}$$

$$y_1=f_{y_1}(x_2,x_3,x_4), \tag{2}$$

$$y_2=f_{y_2}(x_5,x_6,x_7), \tag{3}$$

$$y_3=f_{y_3}(x_8,x_9,x_{10}), \tag{4}$$

where $f(\cdot)$ denotes an input-output mapping in a fuzzy knowledge base form; y_1 is the quality of the brand product; y_2 is the image of the brand product; y_3 is the service connected with the brand product; x_1 is the price of the brand product; x_2 is the quality of the product project; x_3 is the quality of the productive technologies; x_4 is the level of the people ware; x_5 is the enterprise status; x_6 is level of advertising campaign; x_7 is level of the reclamations; x_8 is the level of the selling service; x_9 is the level of the after-sale service; x_{10} is the bonus attached to the brand-product.

The value of a particular factor x_i $(i=\overline{1,10})$ will be determined as a deviation (in percentages) from average figures of rival brand products on the given market.

3 Fuzzy Knowledge Bases

Mamdani-type fuzzy knowledge bases represent the "input-output" ties in (2), (3), and (4). Tables 1–3 show fragments of those bases. There are 49 fuzzy rules in total. The rules are produced a market expert.

Relation (1) is modeled by the following Sugeno-type fuzzy rules:

$$\left.\begin{array}{l}
\textbf{If } x_1 \text{ is High and } y_1 \text{ is Low and } y_2 \text{ is Low and } y_3 \text{ is Low,} \\
\quad \textbf{then } Q=-0.08x_1+0.03y_1+0.025y_2+0.055y_3+14; \\
\textbf{If } x_1 \text{ is Average and } y_1 \text{ is Average and } y_2 \text{ is Average and } y_3 \text{ is Average,} \\
\quad \textbf{then } Q=-0.35x_1+0.4y_1+0.28y_2+0.05y_3+50; \\
\textbf{If } x_1 \text{ is Low and } y_1 \text{ is High and } y_2 \text{ is High and } y_3 \text{ is High,} \\
\quad \textbf{then } Q=-0.06x_1+0.06y_1+0.06y_2+0.08y_3+80.
\end{array}\right\} \tag{5}$$

Each rule in (5) corresponds to one sale strategy. For the first strategy, the values of the price, and the quality, and the image, and the service are bad, regarding a customer's point of view. They are average for the second type of sale strategy, and they are good in case of the third strategy. It is assumed, that an elasticity of the competitive strength index on factor changing is constant inside single sale strategy. Coefficients in rule consequents in (5) correspond to competitive strength sensitivity for relevant factors. They are calculated via Saaty's method [4] on base of paired comparisons the importance of the factors.

Table 1. Fuzzy expert knowledge about relation (2)

x_2	x_3	x_4	y_1
High	High	High	High
High	High	Average	High
High	Average	High	High
Low	Low	Low	Low
Average	Low	Low	Low
Average	Low	Average	Low
High	Average	Low	Average
Low	Average	High	Average
Average	High	Low	Average
Average	Low	High	Average
Average	Average	Average	Average

Table 2. Fuzzy expert knowledge about relation (3)

x_5	x_6	x_7	y_2
High	High	Average	High
Any	High	Low	High
Average	High	Average	High
Any	Low	High	Low
Low	Low	Average	Low
Low	Average	High	Low
High	Low	Average	Average
High	Average	High	Average
Low	High	Average	Average
Average	Average	Average	Average

Table 3. Fuzzy expert knowledge about relation (4)

x_8	x_9	x_{10}	y_3
High	High	High	High
High	High	Average	High
High	Average	High	High
Low	Low	Low	Low
Low	Average	Average	Low
Average	Low	Low	Low
High	Average	Low	Average
Low	High	Average	Average
Low	Average	High	Average
Average	Average	Average	Average

Figure 1 shows membership functions of terms from the fuzzy knowledge bases. The following Gaussian curve membership function is used:

$$\mu^t(x)=\exp\left(-(x-z)^2/(2c^2)\right), \qquad (6)$$

where $\mu^t(x)$ is a membership function of variable x to term t; z is the coordinate of the curve center; c is the coefficient of the curve concentration.

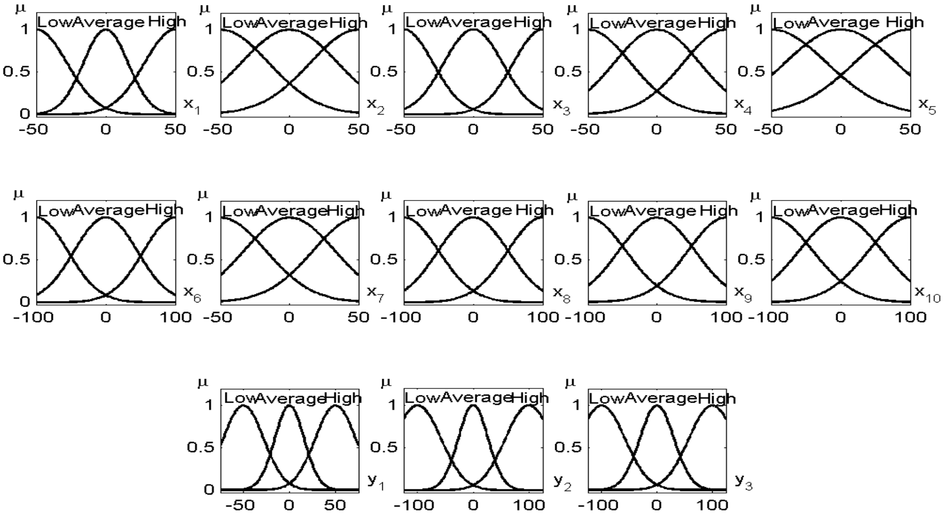

Figure 1. Membership functions

4 Fuzzy Inference

Typical Mamdani-type and Sugeno-type fuzzy inference algorithms are used [2, 3]. They are specified below.

Let us describe a Mamdani-type fuzzy rule base for relation $y=f(x_1,x_2,...,x_n)$ as follows:

$$\text{If } \ x_1=\tilde{a}_1^j \text{ and } x_2=\tilde{a}_2^j \text{ and ... and } x_n=\tilde{a}_n^j, \quad \text{then } \ y=\tilde{d}_j, \quad j=\overline{1,m}, \tag{7}$$

where $\tilde{a}_i^j$ and $\tilde{d}_j$ are fuzzy terms with membership functions $\mu^{a_i^j}(x_i)$ and $\mu^{d_j}(y)$, $x_i \in [\underline{x}_i, \overline{x}_i]$, $y \in [\underline{y}, \overline{y}]$, $i=\overline{1,n}$.

The following fuzzy value of output y corresponds to the given input vector $\mathbf{X}=(x_1,x_2,...,x_n)$:

$$\tilde{y}=\left(\frac{\mu^{d_1}(\mathbf{X})}{\tilde{d}_1},\frac{\mu^{d_2}(\mathbf{X})}{\tilde{d}_2},...,\frac{\mu^{d_m}(\mathbf{X})}{\tilde{d}_m}\right),$$

where $\mu^{d_j}(\mathbf{X})=\mu^{a_1^j}(x_1)\wedge\mu^{a_2^j}(x_2)\wedge...\wedge\mu^{a_n^j}(x_n)$ is a membership grade of vector $\mathbf{X}$ to fuzzy set $\tilde{d}_j$, according to the j-th rule of knowledge base (7); $\wedge$ denotes t-norm (multiplication in this paper).

The membership function of fuzzy number $\tilde{y}$ on support $[\underline{y}, \overline{y}]$ is calculated as follows:

$$\mu_{\tilde{y}}(y)=\max_{j=\overline{1,m}} \ \min\left(\mu^{d_j}(\mathbf{X}), \mu^{d_j}(y)\right), \quad j=\overline{1,m}.$$

The crisp value of $\tilde{y}$ is computed by the following defuzzification:

$$y=\frac{\int_{\underline{y}}^{\overline{y}} y\cdot\mu_{\tilde{y}}(y)dy}{\int_{\underline{y}}^{\overline{y}} \mu_{\tilde{y}}(y)dy}.$$

Let us describe the Sugeno-type fuzzy rule base for relation $y=f(x_1,x_2,...,x_n)$ as follows:

$$\text{If } \ x_1=\tilde{a}_1^j \ \text{ and } \ x_2=\tilde{a}_2^j \ \text{ and ... and } \ x_n=\tilde{a}_n^j, \text{ then } y=d_j, \ \ j=\overline{1,m}, \tag{8}$$

where $d_j=b_0^j+b_1^j\cdot x_1+b_1^j\cdot x_2+...+b_1^j\cdot x_n$ is a linear function; b_p^j is a real number, $p=\overline{0,n}$.

The following output fuzzy value corresponds to $\mathbf{X}=(x_1,x_2,...,x_n)$:

$$\tilde{y}=\left(\frac{\mu^{d_1}(\mathbf{X})}{d_1},\frac{\mu^{d_2}(\mathbf{X})}{d_2},...,\frac{\mu^{d_m}(\mathbf{X})}{d_m}\right). \qquad (9)$$

The crisp value of (9) is calculated by the following defuzzification:

$$y=\frac{\sum_{j=\overline{1,m}}\mu^{d_j}(\mathbf{X})\cdot d_j}{\sum_{j=\overline{1,m}}\mu^{d_j}(\mathbf{X})}. \qquad (10)$$

5 Examples

Task 1. Experts assessed the figures of brand product "Vodka Podilya" (TM "Sotka") on Vinnitsa regional market as follows: x_1=10%, x_2=High, x_3=Average, x_4=Average, x_5=Average, x_6=-50%, x_7=-40%, x_8=-30%, x_9=Average, and x_{10}=-80% (data on September, 2004).

As a result of Mamdani fuzzy inferences we obtain the following values of the enlarged influence factors: y_1=9.15 %, y_2=11.9 %, and y_3=-34.7 %. Figure 2 illustrates those fuzzy inferences. After applying formula (9) and (10) to fuzzy knowledge base (5), the competitive strength index of "Vodka Podilya" is inferred as $Q_P=51.75$.

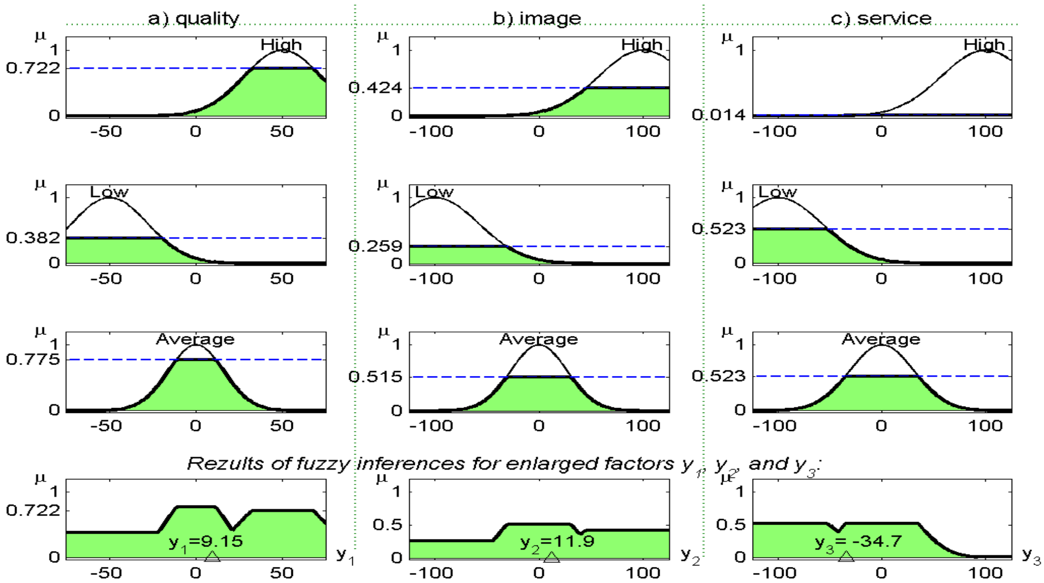

Figure 2. Mamdani-type fuzzy logic inferences for task 1

Task 2. Experts assessed the figures of brand product "Vodka Nemiroff Original" (TM "Nemiroff") on the same market as follows: x_1=40%, x_2=High, x_3=25%, x_4=High, x_5=High, x_6=70%, x_7=-20%, x_8=80%, x_9=Average, and x_{10}=-50% (data on September, 2004).

As a result of the fuzzy modeling we obtain the following value of the competitive strength index of "Vodka Nemiroff Original": $Q_N=62.41$.

Task 3. It is necessary to increase the competitive strength index of "Vodka Podilya" up to $Q^*\geq 63$ that is higher than the rival level $Q_N=62.41$. How to do it with minimal recourses? A manager may change 2 factors in the following frames: $x_1\in[-20,20]$ and $x_6\in[-55,10]$. Increments of factors x_1 and x_2 on 1 request $c_1=-10000$ UAH and $c_6=1500$ UAH.

Figure 3 shows the graphical solution of this optimizing task. 5 dotted lines indicate traces of the goal function with values C=-70000, 0, 100000, 168000, and 300000 UAH. The pentagon points to the optima.

Task 4. How to increase the competitive strength index of "Vodka Podilya" maximally without additional fund? A manager may act as pointed in task 3.

Figure 4 shows the graphical solution of optimizing task 4. The pentagon points to the optima.

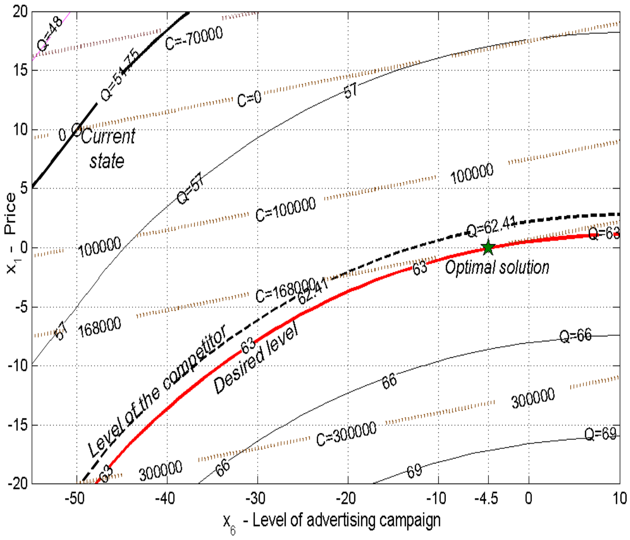

Figure 3. Graphical solving task 3

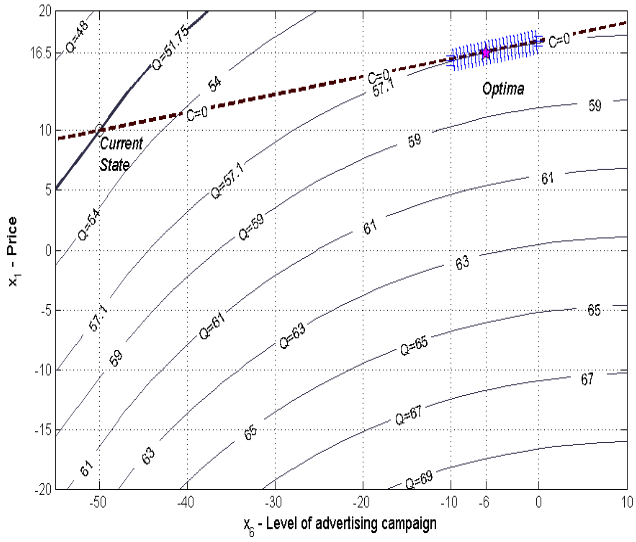

Figure 4. Graphical solving task 4

6 Identification of the Fuzzy Model

The fuzzy model predicte the following values of the competitive strength index: $Q_P=51.75$ for "Vodka Podilya" and $Q_N=62.41$ for "Vodka Nemiroff Original". Hence, a ratio of their predicted market segments β_P and β_N is $\beta_N:\beta_P=Q_N:Q_P=62.41:51.75=1.21:1$. Cashbooks of two Vinnitsa supermarkets "Da! Market" indicate this ratio as 1.29:1 and 1.41:1. There is a difference between actual and inferred results.

An improvement of the fuzzy model may be done by a training the membership functions and the consequents of fuzzy rules. Let's us define the training set as follows:

$$(\mathbf{X}_{ij},\beta_{ij}),\quad i=\overline{1,H},\quad j=\overline{1,M_i}, \tag{11}$$

where H denotes the number of the regional markets; M_i denotes the number of rival brand products on the i-th market; β_{ij} denotes the actual segment of the i-th market for the j-th brand product; $\mathbf{X}_{ij}$ denotes the vector on the influence factors for the j-th brand product on i-th market.

According to theory of fuzzy identification [3, 5] the training is searching the vector of model parameters (**P**) that minimize the difference between actual and predicted market segmentation:

$$\sqrt{\frac{1}{S}\sum_{i=\overline{1,H}}\sum_{j=\overline{1,M_i}}\left(\beta_{ij}-\beta_F(\mathbf{X}_{ij},\mathbf{P})\right)^2}\to\min,$$

where $\beta_F(\mathbf{X}_{ij},\mathbf{P})$ denotes the predicted market segment for product $\mathbf{X}_{ij}$; $S=\sum_{i=\overline{1,H}}M_i$ is a length of (11).

7 Conclusion

A fuzzy model of brand competitiveness index is proposed. The fuzzy model is based on 4 expert knowledge bases. The model can be the basis for solving various tasks of optimal management of brand products. An example of achievement the desired level of competitive strength with minimal expenses is discussed.

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