Computational Logic and Cognitive Science: An Overview

Session 4: Concepts of Default Reasoning and Non-Monotonicity

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Helmar Gust & Kai-Uwe Kühnberger
University of Osnabrück
Overview

- Non-Monotonicity: The Problem
- Reiter’s Default Logic
- Inheritance Networks
- Preferred Models
"... default reasoning (unlike logic and arithmetic) is psychologistic in nature; it is by definition the study of how people perform on certain types of problems and in certain types of situations. ..."

Recall Inconsistency in Classical Logic

- If \( A = \{A_1, \ldots, A_n\} \) is inconsistent or unsatisfiable then \( A \) cannot have a model.

- A consequence of this fact is that
  \[ A \vDash \varphi \]
  for every arbitrary \( \varphi \).

- So with a complete calculus we get
  \[ A \vdash \varphi \]
  for any \( \varphi \).

- With a sound calculus we can check consistency by
  \[ A \vdash \varphi \land \neg \varphi \]

- Classical logic cannot handle situations of inconsistency.
Non-Monotonicity: The Problem

- If something is proven in a logical system, then evidence for the contrary cannot change this fact.
- Monotonicity of classical logic:
  - Construction of truth predicates was based on a monotonic evaluation.
  - $X \subseteq Y \rightarrow \text{Th}(X) \subseteq \text{Th}(Y)$.
  - If $\Delta \vdash \varphi$, then $\Delta, \neg \varphi \vdash \varphi$, although we have evidence for the contrary of $\varphi$.
- Non-monotonic logics.
  - A family of formal frameworks designed to capture inferences of common-sense reasoning.
  - Reasoners draw conclusions tentatively, reserving the right to retract them in the light of further information.
  - Non-Monotonicity: Inference can be “undone” or “blocked” by new information.
Non-Monotonicity

Axioms

Birds can *usually* fly.
Penguins are birds.
Tweety is a Penguin.

Theorem

Tweety can fly.

Axioms

Birds can *usually* fly.
Penguins are birds.
Tweety is a Penguin.
Penguins can’t fly.

Theorem

Tweety cannot fly.
Non-Monotonicity

Theorems

Axioms

monotonic extension

Axioms

Theorems without p

new theorems because of + p

Theorems without p

+ p

Axioms

Theorems incl. p

+ p

non-monotonic extension
The non-monotonicity problem is a problem with a long history in AI. Over the years many theories were developed in order to model non-monotonic reasoning:

- **Closed World Assumption**
  - Negation as failure / answer sets / minimal models
  - Intuition: Everything that is not known is false

- **Default logic**
  - Intuition: There is background knowledge and also revisable information

- **Circumscription**
  - Consider only minimal extensions of certain predicates

- **Modal logics**
  - Intuition: In other worlds other things are true

- **Paraconsistent logics**
  - Even contradictory facts do not make the logic globally inconsistent

- **Defeasible logics**

- Etc.
Frameworks for Non-Monotonicity

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    - Default logic
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    - Circumscription
      - Consider only minimal extensions of certain predicates
    - Modal logics
      - Intuition: In other worlds other things are true
    - Paraconsistent logics
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    - Defeasible logics
    - Etc.
The non-monotonicity problem is a problem with a long history in AI. Over the years, many theories were developed in order to model non-monotonic reasoning.

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- **Default logic**
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- **Circumscription**
  - Consider only minimal extensions of certain predicates

- **Modal logics**
  - Intuition: In other worlds, other things are true

- **Paraconsistent logics**
  - Even contradictory facts do not make the logic globally inconsistent

- **Defeasible logics**

- Etc.

**Example:**

\[
\begin{align*}
b(tweety) & \iff b(X), \text{ not } ef(X) \\
\neg f(X) & \iff p(tweety) \\
ef(X) & \iff p(X)
\end{align*}
\]
Reiter’s Default Logic

- A default theory is a pair $<$W,$\Delta$$>$:
  - $W$ is a “world description”, a set of first-order formulas representing the “strict” or background information
  - $\Delta$ is a set of defaults, representing revisable information
  - Inference rules in $\Delta$ have the form: $\gamma : \theta \vdash \tau$
    where $\gamma$, $\theta$, $\tau$ are also first-order formulas with
    - $\gamma$: prerequisites (represented as a conjunction of literals)
    - $\theta$: justification / consistency assumptions
    - $\tau$: default (revisable conclusion)
  - Interpretation: If $\gamma$ is known and if there is no evidence that $\theta$ might be false, then $\tau$ can be derived.
  - If $\theta = \tau$ we call this a normal default
Reiter’s Default Logic

- Tweety the penguin:
  - The world description $W$ and the default set $\Delta$
    - $W = \{\text{bird(Tweety)}, \text{bird(Paul)}, \text{penguin(Tweety)}, \forall x: \text{penguin}(x) \rightarrow \neg \text{flies}(x)\}$
    - $\Delta = \{\forall x: \text{bird}(x) : \text{flies}(x) \lor \neg \text{flies}(x)\}$
  - From $\text{bird(Paul)}$ we can derive $\text{flies(Paul)}$
    - There is no evidence that Paul cannot fly
  - From $\text{bird(Tweety)}$ we cannot derive $\text{flies(Tweety)}$ because there is evidence that $\neg \text{flies(Tweety)}$
    - Using $\text{penguin(Tweety)}, \text{penguin(Tweety)} \rightarrow \neg \text{flies(Tweety)}$
      - we can infer that $\neg \text{flies(Tweety)}$
      - by Modus Ponens
Reiter’s Default Logic

- The Nixon problem:
  - Nixon is a Quaker
  - Nixon is a Republican
  - Quakers are pacifists
  - Republicans are not pacifists
  - Question: Is Nixon a pacifist?
Reiter’s Default Logic

The Nixon problem:
- $W = \{\text{Nixon}, \text{Nixon} \rightarrow \text{Republican}, \text{Nixon} \rightarrow \text{Quaker}\}$
- $\Delta = \{\text{Republican} : \neg \text{Pacifist} \lor \neg \text{Pacifist}, \text{Quaker} : \text{Pacifist} \land \text{Pacifist}\}$

Questions:
- Can we derive $\neg \text{Pacifist}$
- Can we derive $\text{Pacifist}$
- Can we derive $\text{Pacifist} \land \neg \text{Pacifist}$
Inheritance Networks
Inheritance Networks

- For Concepts $A$, $B$, $C$ the diagram

  - $A$s are (usually) $B$s (arc with polarity +)
  - $C$s are (usually) not $B$s (arc with polarity -)

- More abstract: The network defines a subsumptions relation
  - $B$ subsumes $A$
  - The complement of $B$ subsumes $C$

- Arcs encode explicit knowledge
Inheritance Networks

- Direct contradictions are not allowed:

- These two arcs represent a direct contradiction in the explicit knowledge
- We do not allow parallel arcs with different polarity
Inheritance Networks

- The Nixon diamond:
  - Nixon is a Quaker
  - Nixon is a Republican
  - Quakers are (usually) pacifists
  - Republicans are (usually) not pacifists

Question: Is Nixon a pacifist?

The intended answer is:
- Don’t know or Maybe
The Nixon diamond:

- This again is a contradiction, but not a direct one.
- In classical logic such a contradiction has the effect that we can derive everything.
- What we want is to keep that Nixon is a Quaker and a Republican, but we want to prohibit to derive both,
  - that Nixon is a pacifist and
  - that Nixon is not a pacifist.
Inheritance Networks

- Tweety the penguin:

means:
- Tweety is a penguin
- Tweety is a bird
- Penguins are birds
- Birds can fly
- Penguins cannot fly

- Question: Can Tweety fly?
- The intended answer is: No
Inheritance Networks

- Valid paths:
  - $B$ subsumes $A$ if there is a valid path from $A$ to $B$.

- **Definition: Valid Path**
  - For every node $X$, the empty path $+[X]$ is valid
  - A path $P=p[X_1, \ldots, X_n]$ is valid if
    - $+[X_1, \ldots, X_{n-1}]$ is valid (upward chaining)
    - there is an edge with polarity $p$ from $X_{n-1}$ to $X_n$
    - $P$ is not precluded
Inheritance Networks

- **Preclution**

  ![Diagram showing a precluded path with nodes X_1, X_k, ..., X_n and a node Y.]

- **Definition:**
  - A path $P = p[X_1, ..., X_k, ..., X_n]$ is precluded if
    - there is a node $Y$
    - a valid path $+[X_1, ..., Y]$
    - a valid path $+[Y, ..., X_k]$ (split path validity)
    - and an arc from $Y$ to $X_n$ with opposite polarity to $p$
Inheritance Networks

- **Reasoning policies**
  - Direct skepticism (DS)
    - Nodes to which we get valid paths with different polarity are ruled out
  - Intersection of consequences (IC)
    - Check if all valid paths have the same polarity (the same consequence)

- For the Tweety diagram both policies give the intended answer: *No*
  - Since there is only a negative valid path
- For the Nixon diamond (a positive and a negative valid path)
  - Direct skepticism gives the answer: *Don’t know*
  - The answer of *intersection of consequences* can be interpreted as *maybe*
Inheritance Networks

- Reasoning by valid paths:

- valid paths
  - Quaker
  - Pacifist
  - Nixon
  - Republican

- CanFly
  - Bird
  - Penguin
  - Tweety

- preclusion
Inheritance Networks

- A more complex example:

- Is a subsumed by c?
Inheritance Networks

- A more complex example:

- Is a subsumed by c?
- Answer:
  - DS: no
  - IC: no

- direct skepticism
- intersection of consequences
- preclusion
Inheritance Networks

- A more complex example:

- Is a subsumed by c?
Inheritance Networks

- A more complex example:

- Is a subsumed by c?
  - DC: no
  - IC: maybe

- Direct skepticism
- Intersection of consequences
- Preclusion
Preferred Models
Preferred Models

- Given a logical language \( \mathcal{L} \)
- \( M^{\mathcal{L}} \) the set of possible models of a logical language \( \mathcal{L} \)
- \( M : \wp(\mathcal{L}) \rightarrow \wp(M^{\mathcal{L}}) \) assigns the set of models to a set of formulas
- \( \mu : \wp(M^{\mathcal{L}}) \rightarrow \wp(M^{\mathcal{L}}) \) maps sets of models to sets of preferred models
- In addition to the classical semantics we can define: 
  
  **Nonmonotonic Entailment**
  
  - \( T \models \phi \Leftrightarrow \mu(M(T)) \subseteq M(\phi) \)
  - where \( T \) is a set of formulas
Preferred Models

- We can define preferred models by a preference relations on (possible) models:
  - $< \subseteq M^\mathcal{L} \times M^\mathcal{L}$
  - $<$ can be an arbitrary antisymmetric relation

- Preferred models as minimal models (according to $<$)

**Definition: Minimal Models**

- $\mu(X) := \{m \in X \mid \neg \exists m' \in X : m' < m\}$
Preferred Models

- Tweety the penguin
  - Tweety $\rightarrow$ Penguin
  - Penguin $\rightarrow$ Bird
  - Bird $\rightarrow$ CanFly
  - Penguin $\neg$ CanFly
Preferred Models

- The Nixon problem
  - Nixon → Quaker
  - Nixon → Republican
  - Quaker → Pacifist
  - Republican → ¬ Pacifist
Preferred Models

- If we have a syntactic definition of $\vdash$ by rules of a calculus we can define the Nonmonotonic Closure
  \[ \overline{T} := \{ \varphi \mid T \vdash \varphi \} \]

- In this case we get as preferred Models
  \[ \mu(M(T)) := M(\overline{T}) \]
Preferred Models

- Example:
  \[ PV = \{ \text{Bird, Mammal, CanFly, LayEggs, HasFeathers, CanSwim, Penguin, Tweety} \} \]

- \[ T = \{ \text{Bird} \not\rightarrow \text{CanFly} \land \text{LayEggs} \]
  \[ \text{Bird} \rightarrow \neg \text{Mammal}, \]
  \[ \text{Penguin} \rightarrow \text{Bird} \land \neg \text{CanFly} \land \text{CanSwim}, \]
  \[ \text{Tweety} \rightarrow \text{Penguin} \} \]

- We define:
  \[ T |\sim \varphi \Leftrightarrow \]
  \[ T |\varphi \]
  \[ \lor \exists \alpha : T |\sim \alpha \]
  \[ \land T |\sim \alpha \rightarrow \varphi \]
  \[ \land T |\neg \neg \varphi \]

- \[ \alpha \rightarrow \varphi \land \psi |\sim \alpha \rightarrow \varphi \]

- \[ \mu(M(T)) := M(\overline{T}) \]
References

- Parts can be found at: [www.lif.univ-mrs.fr/~ks/publ.html](http://www.lif.univ-mrs.fr/~ks/publ.html)
More introductory books:

- Aldonelli: Grounded Consequence for Defeasible Logic. Cambridge 2005
More advanced books:
Thank you very much!!