OWL 2 Profiles

An Introduction to Lightweight Ontology Languages

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Remark for the Online Version

This is the online version of the slideset used at the ICCL Summer School 2013 in Dresden (4 x 60min).

Detailed lecture notes that explain the content (and some more) are available online at http://korrekt.org/page/OWL_2_Profiles. These notes also give many pointers to further reading.

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Plan

**Goal:**
Learn about the lightweight ontology languages, and understand why they are lightweight.

1. Review: OWL and DLs
2. Overview: OWL 2 Profiles
3. Reasoning in OWL RL: Instance Retrieval
4. Reasoning in OWL EL: Classification
5. Reasoning in OWL QL: Query Answering
6. Limits of Tractability
7. Advanced Features
The OWL Language

- W3C standard since 2004, updated in 2009

- An ontology language with two sides:
  - Descriptive: express expert knowledge formally
  - Logical: draw conclusions from this knowledge → reasoning

- Compatibility with important technology standards
  - Unicode, IRI, XML Schema, RDF, RDF Schema
The Data Model of OWL

- Ontologies use a vocabulary of entities
  - Classes
  - Properties
  - Individuals and data literals

- Entities are combined to form expressions

- Relationships of expressions are described by axioms
Syntaxes and Semantics

- 5 official syntactic formats:
  - Functional-Style Syntax
  - Manchester Syntax
  - OWL/XML Syntax
  - RDF-based syntaxes (RDF/XML, Turtle)

- 2 formal semantics:
  - Direct Semantics
  - RDF-Based Semantics
Reasoning Tasks

- Every ontology has infinitely many conclusions
- Which conclusions are we interested in?
  - Instance checking
  - Class subsumption
  - Ontology consistency
  - Class consistency (coherence)

→ Tasks can be reduced to each other with little effort
Hardness of Reasoning

- **Main requirements:**
  - Soundness: only correct conclusions are computed
  - Completeness: no correct conclusion is missed

- **Reasoning is hard:**
  - Undecidable for RDF-Based Semantics
  - N2ExpTime-complete for Direct Semantics

→ OWL Profiles: sub-languages with easier reasoning
## OWL and Description Logics: Basic Features

<table>
<thead>
<tr>
<th></th>
<th>OWL Functional-Style Syntax</th>
<th>DL Syntax</th>
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</thead>
<tbody>
<tr>
<td><strong>Axioms</strong></td>
<td>SubClassOf( ( C ), ( D ) )</td>
<td>( C \sqsubseteq D )</td>
</tr>
<tr>
<td></td>
<td>ClassAssertion( ( C ), ( a ) )</td>
<td>( C(a) )</td>
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<td>ObjectPropertyAssertion( ( P ), ( a ), ( b ) )</td>
<td>( P(a, b) )</td>
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<tr>
<td><strong>Class expressions</strong></td>
<td>ObjectIntersectionOf( ( C ), ( D ) )</td>
<td>( C \sqcap D )</td>
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<td>ObjectUnionOf( ( C ), ( D ) )</td>
<td>( C \sqcup D )</td>
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<td>ObjectComplementOf( ( C ) )</td>
<td>( \neg C )</td>
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<td>owl:Thing</td>
<td>( \top )</td>
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<td>owl:Nothing</td>
<td>( \bot )</td>
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<td>ObjectSomeValuesFrom( ( P ), ( C ) )</td>
<td>( \exists P.C )</td>
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<td>ObjectAllValuesFrom( ( P ), ( C ) )</td>
<td>( \forall P.C )</td>
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<tr>
<td><strong>Property expressions</strong></td>
<td>ObjectInverseOf( ( P ) )</td>
<td>( P^- )</td>
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</tbody>
</table>
Example Axioms in DL Syntax

FelisCatus(silvester)
preysOn(silvester, tweety)

FelisCatus ⊑ Mammalia
∃preysOn.⊤ ⊑ Predator
⊤ ⊑ ∀preysOn.Animal

Animal ⊓ PlaysChess ⊑ HomoSapiens

Mammalia ⊑ ∃hasFather.Mammalia

Silvester is a cat.
Silvester preys on Tweety.
Cats are mammals.
What preys on something is a predator.
What is preyed on is an animal.
All animals that play chess are humans.
Every mammal has a mammal father.
OWL Direct Semantics

- Based on first-order logic interpretations
- Direct correspondences:
  - classes → sets
  - properties → relations
  - individuals → domain elements
- Equivalent to translating OWL to first-order logic
## OWL Direct Semantics

<table>
<thead>
<tr>
<th>Expression $ex$</th>
<th>Interpretation $ex^I$</th>
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<tbody>
<tr>
<td>$C \cap D$</td>
<td>$C^I \cap D^I$</td>
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<td>$C \cup D$</td>
<td>$C^I \cup D^I$</td>
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<tr>
<td>$\neg C$</td>
<td>$\Delta^I \setminus C^I$</td>
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<tr>
<td>$\top$</td>
<td>$\Delta^I$</td>
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<tr>
<td>$\bot$</td>
<td>$\emptyset$</td>
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<tr>
<td>$\exists P.C$</td>
<td>${ e \mid \text{there is } f \text{ with } \langle e, f \rangle \in P^I \text{ and } f \in C^I }$</td>
</tr>
<tr>
<td>$\forall P.C$</td>
<td>${ e \mid \text{for all } f \text{ with } \langle e, f \rangle \in P^I \text{ we have } f \in C^I }$</td>
</tr>
<tr>
<td>$P^-$</td>
<td>${ \langle f, e \rangle \mid \langle e, f \rangle \in P^I }$</td>
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</tbody>
</table>
OWL RDF-Based Semantics

- Based on translating OWL ontologies to RDF graphs
- Interpretations defined on graphs
  - Applicable to all RDF graphs, even if not from OWL
- Sometimes stronger (more entailments), sometimes weaker (fewer entailments) than Direct Semantics
- Direct Semantics and RDF-Based Semantics agree under reasonable conditions
Reasoning in the OWL Profiles
Defining Language Profiles by Grammars

- The OWL sublanguage introduced above is ALCI
- Can be described by a formal grammar:

\[
\text{ALCI}
\]

\[
\text{Axiom ::= } C \subseteq C \mid C(\text{IName}) \mid P(\text{IName, IName})
\]

\[
C ::= \text{CName} \mid \top \mid \bot \mid C \cap C \mid C \cup C \mid \neg C \mid \exists P.C \mid \forall P.C
\]

\[
P ::= \text{PName} \mid \text{PName}^{-}
\]
A Tiny Version of OWL EL

- OWL EL is based on the description logic EL(++)
- Typical applications in ontology engineering

\[ \mathcal{EL}_{tiny} \]

\[
\text{Axiom ::= } C \sqsubseteq C \mid C(\text{IName}) \mid P(\text{IName}, \text{IName}) \\
C ::= \text{CName} \mid \top \mid \bot \mid C \sqcap C \mid \exists P.C \\
P ::= \text{PName}
\]
A Tiny Version of OWL RL

- OWL RL is a “rule language”
- Typical applications in data management

\[\mathcal{RL}_{tiny}\]

\[
\text{Axiom} ::= \text{CL} \subseteq \text{CR} \mid \text{CR(IName)} \mid \text{P(IName, IName)} \\
\text{CL} ::= \text{CName} \mid \bot \mid \text{CL} \sqcap \text{CL} \mid \text{CL} \sqcup \text{CL} \mid \exists \text{P.CL} \\
\text{CR} ::= \text{CName} \mid \bot \mid \text{CR} \sqcap \text{CR} \mid \neg \text{CL} \mid \forall \text{P.CR} \\
\text{P} ::= \text{PName} \mid \text{PName}^\sim
\]
A Tiny Version of OWL QL

- OWL QL is a “query language”
- Typical applications in data access

\textit{QL_{tiny}}

\begin{align*}
\text{Axiom} &::= \text{CL} \sqsubseteq \text{CR} \mid \text{CR(IName)} \mid \text{P(IName, IName)} \\
\text{CL} &::= \text{CName} \mid \top \mid \bot \mid \exists \text{P.} \top \\
\text{CR} &::= \text{CName} \mid \top \mid \bot \mid \text{CR} \sqcap \text{CR} \mid \neg \text{CL} \mid \exists \text{P.CR} \\
\text{P} &::= \text{PName} \mid \text{PName}^-
\end{align*}
Tiny OWL Profiles: Feature Overview

- RL and QL allow for inverse properties, EL doesn't.
- Features for sub- and superclasses:

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Rule-Based Instance Retrieval in RL
Rule-Based Instance Retrieval in RL

- **Goal:** for an ontology $O$, compute all entailments of the form $C(a)$ and $P(a,b)$ for a class name $C$ in $O$ and a property name $P$ in $O$

- **Approach:** apply inference rules until no new conclusions are found

- Known under many names: saturation, deductive closure, materialisation, bottom-up reasoning, forward chaining, consequence-based reasoning
Derivation Rules for RL

\[ \text{A}_{\equiv} \quad \frac{D(c)}{E(c)} : D \sqsubseteq E \in O \]

\[ \text{A}_{\cap}^- \quad \frac{D_1 \cap D_2(c)}{D_1(c)} \quad \frac{D_2(c)}{D_1 \cap D_2(c)} \]

\[ \text{A}_{\forall}^- \quad \frac{\forall P.E(c)}{P(c, d)} \quad \frac{P(c, d)}{E(d)} \]

\[ \text{A}_{\neg}^- \quad \frac{\neg D(c)}{D(c)} \quad \frac{D(c)}{\neg(c)} \]

\[ \text{A}_{\text{inv}}^- \quad \frac{P^-(c, d)}{P(d, c)} \]

\[ \text{A}^+ \quad \frac{D_1(c) \quad D_2(c)}{D_1 \cap D_2(c)} : D_1 \cap D_2 \text{ occurs in } O \]

\[ \text{A}_{\exists}^+ \quad \frac{P(c, d)}{\exists P.E(c)} \quad \frac{E(d)}{\exists P.E(c)} \]

\[ \text{A}_{\sqcup}^+ \quad \frac{D(c)}{D_1 \sqcup D_2(c)} : D = D_1 \text{ or } D = D_2 \]

\[ \text{A}_{\text{inv}}^+ \quad \frac{P(c, d)}{P^-(d, c)} : P^- \text{ occurs in } O \]
Derivation Calculus

- Saturate under the derivation rules (this is uniquely defined: Section 3.3 lecture notes)

- An axiom is inferred if
  - the axiom was derived by the rules, or
  - ⊥(c) was derived for some constant c.

→ Second case takes inconsistent ontologies into account
Example Derivation for RL

FelisCatus $\sqsubseteq \forall \text{preysOn.}(\text{Animal} \sqcap \text{Small})$
Animal $\sqcap \exists \text{preysOn.}\text{Animal} \sqsubseteq \text{Predator}$
FelisCatus $\sqsubseteq \text{Animal}$
FelisCatus(silvester)
preysOn(silvester, tweety)

\[
\begin{align*}
A_\sqsubseteq & \quad \frac{D(c)}{E(c)} : D \sqsubseteq E \in O \\
A_\sqcap^- & \quad \frac{D_1 \sqcap D_2(c)}{D_1(c) \quad D_2(c)} \\
A_\forall^- & \quad \frac{\forall P.E(c) \quad P(c, d)}{E(d)} \\
A_\sqcap^+ & \quad \frac{D_1(c) \quad D_2(c)}{D_1 \sqcap D_2(c)} : D_1 \sqcap D_2 \text{ occurs} \\
A_\exists^+ & \quad \frac{P(c, d) \quad E(d)}{\exists P.E(c)} : \exists P.E \text{ occurs}
\end{align*}
\]
Example Derivation for RL

FelisCatus $\sqsubseteq \forall \text{preysOn.}(\text{Animal } \sqcap \text{Small})$

Animal $\sqcap \exists \text{preysOn.} \text{Animal} + \text{Predator}$

FelisCatus $\sqsubseteq \text{Animal}$

FelisCatus(silvester)

preysOn(silvester, tweety)

Animal(silvester)

$\forall \text{preysOn.}(\text{Animal } \sqcap \text{Small})(\text{silvester})$

Animal $\sqcap \text{Small}(\text{tweety})$

Animal(tweety)

Small(tweety)

$\exists \text{preysOn.} \text{Animal(silvester)}$

Animal $\sqcap \exists \text{preysOn.} \text{Animal(silvester)}$

Predator(silvester)
Correctness of the RL Rule Calculus
Correctness of the RL Rule Calculus

Soundness

Completeness

Termination
Soundness of the Calculus
Soundness of the Calculus

- Proof strategy:
  - Show that every single rule is sound
  - If we start with true statements, only true statements can be derived (that's an induction argument)
  - Easy to see
Termination of the Calculus
Termination of the Calculus

- Proof strategy:
  - Show that only a limited number of inferences can be derived
  - Main observation: every derived axiom only uses expressions from the ontology (or $\bot$)
  - Only finite number of axioms possible (at most size$^3$ many)
Completeness of the Calculus (for instance retrieval!)
Completeness of the Calculus
(for instance retrieval!)

- Proof strategy:
  - Show that, if a axiom is not inferred, then there is a model of O here the axiom does not hold
  - There even is a single \textit{universal model} that refutes every axiom that is not inferred

- Proof steps:
  - Define this model
  - Show that it is a model
  - Show that it refutes non-inferred axioms
Defining a Universal Model

- Let $O'$ be the saturation of $O$.
- We define an interpretation $I$:
  - The domain $\Delta^I$ of $I$ is the set of all individual symbols (w.l.o.g., we can assume that there is one).
  - For every individual symbol $c$, define $c^I := c$.
  - For every class name $A$, define $c \in A^I$ iff $A(c) \in O'$.
  - For every property name $P$, define $<c, d> \in P^I$ iff $P(c, d) \in O'$.

$\rightarrow I$ refutes atomic assertions that are not in $O'$.
Example Derivation for RL

FelisCatus ⊑ ∀preysOn.(Animal ⊓ Small)
Animal ⊓ ∀preysOn.Animal ⊑ Predator
FelisCatus ⊑ Animal
FelisCatus(silvester)
preysOn(silvester, tweety)

Animal(silvester)
∀preysOn.(Animal ⊓ Small)(silvester)
Animal ⊓ Small(tweety)
Animal(tweety)
Small(tweety)
∃preysOn.Animal(silvester)
Animal ⊓ ∃preysOn.Animal(silvester)
Predator(silvester)
Constructing a Universal Model

FelisCatus(silvester)
preysOn(silvester, tweety)
Animal(silvester)

Animal(tweety)
Small(tweety)

Predator(silvester)
Constructing a Universal Model

FelisCatus(silvester)
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Animal(tweety)
Small(tweety)

Predator(silvester)
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Small(tweety)

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Constructing a Universal Model

FelisCatus ⊑ ∀ preysOn.(Animal ⋂ Small)
Animal ⋂ ∃ preysOn.Animal ⊑ Predator
FelisCatus ⊑ Animal
FelisCatus(silvester)
preysOn(silvester, tweety)

Animal(silvester)
∀ preysOn.(Animal ⋂ Small)(silvester)
Animal ⋂ Small(tweety)
Animal(tweety)
Small(tweety)
∃ preysOn.Animal(silvester)
Animal ⋂ ∃ preysOn.Animal(silvester)
Predator(silvester)
Completing the Completeness Proof (1)

- I and O' agree on class and property names.
- Extend this to complex expressions:
  1. $P^-$ occurs in O and $\langle c, d \rangle \in P^{-1}$ iff $P^-(c, d) \in O'$
  2. If $E \in CL$ occurs in O, then $c \in E^I$ implies $E(c) \in O'$
  3. If $E \in CR$ and $E(c) \in O'$, then $E$ occurs in O and $c \in E^I$

- Easy consequence: I satisfies O
Completing the Completeness Proof (2)

- **Example Claim 2:** If $E \in \text{CL}$ occurs in $O$, then $c \in E^I$ implies $E(c) \in O'$
- **Proof technique:** structural induction on the grammatical definition of CL

$$\text{CL} ::= \text{CName} \mid \bot \mid \text{CL} \sqcap \text{CL} \mid \text{CL} \sqcup \text{CL} \mid \exists P. \text{CL}$$

- Not hard in each case; for example:

Case $E = D_1 \sqcap D_2$. By the semantics of $\sqcap$, we find $c \in D_1^I$ and $c \in D_2^I$. Clearly, $D_1$ and $D_2$ occur in $O$ since $E$ does. Thus, the induction hypothesis implies $D_1(c) \in O'$ and $D_2(c) \in O'$. Since $E$ occurs in $O$, rule $A^+_{\sqcap}$ applies and $E(c) \in O'$ as required.
Name: ellipsoid body

Definition: A doughnut shaped synaptic neuropil domain of the central body complex of the adult brain that lies just anterior to the fan-shaped body. Its hole (the ellipsoid body canal) points anteriorly and has an axon tract (the anterior bundle) running through it. It is divided into concentric layers and into 16 radial segments, 8 per hemisphere.

Synonyms:
* EB
* eb

Relationships:
* part_of central body

Check In FlyBase >>
Rule-Based Classification in EL

- **Goal:** for an ontology O, compute all entailments of the form \( A \sqsubseteq B \) for class names A and B in O.

\[
\rightarrow \text{Possible with similar inference rules as for RL}
\]
Derivation Rules for EL

- Ignore assertions: assume we only have class inclusions here

\[
\begin{align*}
T_{\subseteq}^+ & : C \subseteq D \quad C \subseteq E \quad D \subseteq E \in O \\
T_{\cap}^- & : C \subseteq D_1 \cap D_2 \\
T_{\exists}^- & : C \subseteq \exists P. \bot \\
T_{i}^+ & : C \subseteq C \\
T_{\cap}^+ & : C \subseteq D_1 \cap D_2 \\
T_{\exists}^+ & : C \subseteq \exists P. D \\
T_{\exists}^+ & : C \subseteq \exists P. E
\end{align*}
\]
Derivation Calculus

- Saturate under the derivation rules

- An axiom $A \sqsubseteq B$ is inferred if
  - $A \sqsubseteq B$ was derived by the rules, or
  - $A \sqsubseteq \bot$ was derived, or
  - $\top \sqsubseteq \bot$ was derived

→ Second case takes inconsistent class into account
→ Third case takes inconsistent ontology into account
Correctness of the EL Rule Calculus

Soundness

Completeness

Termination
Correctness of the EL Rule Calculus

- Soundness and termination as for RL

- Completeness similar to RL, but with different model $I$:
  - Introduce one representative domain element $e_C$ for every class name $C$ that is not inconsistent.
  - For class name $A$, define $e_C \in A^I$ iff $C \subseteq A \in O'$.
  - For property name $P$, define $<e_C, e_D> \in P^I$ iff $C \subseteq \exists P.D \in O'$.

→ Not a universal model, but a “canonical” one
Towards a Practical Implementation

- Optimisation is essential
- Main aspects of optimisation:
  - Derivation rules: avoid redundant inferences
  - Control flow: avoid unnecessary rule applications
  - Engineering: fast rule matching
  - Concurrency: parallelise computation
- Important insight: main cost is in checking applicability of rules, not in actually applying them
Optimising Derivation Rules

- Avoid inferences that we are not interested in → goal directed reasoning

- Example: further restrict the initialisation rule:

\[
\begin{array}{c}
T^+ \\
C \sqsubseteq C \\
C \sqsubseteq T
\end{array}
\vdash C \text{ occurs in } O
\]
Optimising Derivation Rules

▪ Avoid inferences that we are not interested in
  → goal directed reasoning

▪ Example: further restrict the initialisation rule:

\[
\begin{align*}
T^+ \quad & \\
C \sqsubseteq C & \quad C \sqsubseteq T
\end{align*}
\] : $C$ occurs in $\mathcal{O}$

→ new fact type init($C$) to mark “relevant” classes:

\[
\begin{align*}
\text{init}(C) & \\
C \sqsubseteq C & \\
E \sqsubseteq \exists R.C & \quad \text{init}(C)
\end{align*}
\]
Control Flow: Naive Evaluation

\[
Closure := \{\text{Init}(C) | \text{for each class name C}\} \\
continue := \text{True} \\
\text{while} \ continue: \\
\hspace{1em} continue := \text{False} \\
\text{for} \ R \ \text{in inference rules:} \\
\hspace{1em} \text{for} \ match \ \text{in matches}(R): \\
\hspace{2em} Closure.\text{add(\text{consequences of match})} \\
\hspace{1em} continue := \text{True}
\]
Control Flow: Semi-Naive Evaluation

- Semi-naive evaluation avoids repeated match checks

\[
Closure := \{\}\nTodo := \{|\text{Init}(C)\mid \text{for each class name } C\}\n\textbf{while } Todo \neq \{\} : \\
\text{exp} := Todo.\text{takeNextElement}() \\
\textbf{if } \text{exp} \text{ not in } Closure: \\
\textbf{for } R \text{ in inference rules:} \\
\quad \textbf{for } \text{match} \text{ in matches}(R, \text{exp}): \\
\quad \quad Todo.\text{add}(\text{consequences})
\]
Engineering Optimisations

- Fast implementations needed for:
  
  "for match in matches(R, exp):"

- Use indexes for fast checks of side conditions and premises

- Join-order optimisations
Concurrency

- Modern computers have many CPUs → How can we use them effectively?

- General strategy:
  - Split task into many independent tasks
  - Idea: partition potential rule applications (matches) among parallel workers
Concurrenty

- Possible strategy: workers ↔ inference rules
  - Problem: inherent limit to number of workers

- Better strategy: partition derived facts by “contexts”
  - Idea: one “context” per class expression
  - Axiom $C \sqsubseteq D$ belongs to context for $C$
  - Axiom $C \sqsubseteq \exists P.D$ also belongs to context for $D$

$\rightarrow$ As many contexts as classes
$\rightarrow$ One Todo queue per context
Concurrency

- Every inference still possible “within” one context:

\[
\begin{align*}
T^\leq & \quad \frac{C \sqsubseteq D}{C \sqsubseteq E} : D \sqsubseteq E \in \mathcal{O} \\
T^- & \quad \frac{C \sqsubseteq D_1 \cap D_2}{C \sqsubseteq D_1 \quad C \sqsubseteq D_2} \\
T^- & \quad \frac{C \sqsubseteq \exists P. \bot}{C \sqsubseteq \bot} \\
T^+_i & \quad \frac{C \sqsubseteq C}{C \sqsubseteq C} : C \text{ occurs in } \mathcal{O} \\
T^+_i & \quad \frac{C \sqsubseteq D_1 \quad C \sqsubseteq D_2}{C \sqsubseteq D_1 \cap D_2} : D_1 \cap D_2 \text{ occurs in } \mathcal{O} \\
T^- & \quad \frac{C \sqsubseteq \exists P.D}{C \sqsubseteq \exists P.D} \\
T^+_i & \quad \frac{C \sqsubseteq \exists P.D \quad D \sqsubseteq E}{C \sqsubseteq \exists P.E} : \exists P.E \text{ occurs in } \mathcal{O}
\end{align*}
\]
ELK Reasoner

- Implemented (with further improvements) in ELK:
  
  http://code.google.com/p/elk-reasoner/

- For more optimisations and further details see

  Y. Kazakov, MK, F. Simancik: “The Incredible ELK”
  (available online)
Querying ontologies

- DL is often not enough for data access:
  - “Who lives together with their parents?”
  - “Who has married parents?”
  - “Which properties connect Alice and Bob?”
  → Not expressible in DL

- More advanced query languages can be used, e.g., SPARQL
  → Semantics not always clear
  → Standard reasoning algorithms often not applicable
Ontology-Based Data Access (OBDA)

- The OBDA paradigm:
  - Query data from a database, taking ontological axioms into account
  - Models of ontology+database can be viewed as (possibly very large) completed database
  - Query answers in OBDA: **certain answers** (answers that hold in every model of the ontology)

→ Query answering under constraints (background knowledge), e.g., for data integration tasks
Conjunctive Queries (CQs)

- Most basic query language
- A query is a formula of the form

\[ \exists x_1 \ldots \exists x_n. C_1 \land \ldots \land C_m \]

where \( C_1, \ldots, C_m \) are atomic first-order formulae
(in our case: predicates are only unary [class] or binary [property])

- A query can have free variables, called distinguished
- The bound variables \( x_1, \ldots, x_n \) are non-distinguished
Conjunctive Queries: Examples

- Who lives together with their parents?
  \( \exists y. \text{livesWith}(x,y) \land \text{hasParent}(x,y) \)

- Who has married parents?
  \( \exists y. \exists z. \text{hasMother}(x,y) \land \text{hasFather}(x,z) \land \text{married}(y,z) \)
Conjunctive Queries: Relationships

- CQs are essentially equivalent to:
  - Basic Graph Patterns of SPARQL
  - SELECT-PROJECT-JOIN fragment of SQL

→ Definition of distinguished/non-distinguished variables must be clarified in either case

- Unions of CQs (UCQs): allow disjunctions of CQs
- Path queries: allow regular expressions over properties
CQs and OBDA

- Challenge:
  Existence of unnamed individuals may be entailed
  → Can we query for these? How?

- Solution:
  Distinguished variables can only bind to named ind's
  Non-distinguished variables can bind to anything
Example

- Query: \(\text{FelisCatus}(x) \land \text{preysOn}(x,y) \land \text{Animal}(y)\)

\[
\begin{align*}
\text{FelisCatus}(\text{silvester}) \\
\text{FelisCatus}(\text{tom}) \\
\text{SerinusCanaria}(\text{tweety}) \\
\text{preysOn}(\text{silvester}, \text{tweety}) \\
\text{SerinusCanaria} \sqsubseteq \text{Animal} \\
\text{FelisCatus} \sqsubseteq \exists \text{preysOn.Animal}
\end{align*}
\]
Example

- Query: \( \text{FelisCatus}(x) \land \text{preysOn}(x,y) \land \text{Animal}(y) \)

\[
\begin{align*}
\text{FelisCatus}(\text{silvester}) \\
\text{FelisCatus}(\text{tom}) \\
\text{SerinusCanaria}(\text{tweety}) \\
\text{preysOn}(\text{silvester}, \text{tweety}) \\
\text{SerinusCanaria} \sqsubseteq \text{Animal} \\
\text{FelisCatus} \sqsubseteq \exists \text{preysOn. Animal}
\end{align*}
\]

→ Only solution is \( x=\text{silvester}, y=\text{tweety} \)

→ No solution with \( x=\text{tom} \) (no named value for \( y \))
Rewriting-Based Query Answering in QL

- **Goal:** for an ontology O, compute all certain answers for a conjunctive query over O

- **Approach:** rewrite input query into a union of many conjunctive queries that yield the result
  - Rewriting only depends on terminological axioms, not on assertions
  - Rewritten queries can be answered by relational DB systems (SQL)
QL Syntax Revisited

QL tiny

Axiom ::= CL ⊆ CR | CR(IName) | P(IName, IName)
CL ::= CName | ⊤ | ⊥ | ∃P.⊤
CR ::= CName | ⊤ | ⊥ | CR ⊓ CR | ¬CL | ∃P.CR
P ::= PName | PName\̄
A Simpler Normal Form for Axioms

- Axioms of QLtiny can be rewritten to a simpler form:

QL normal form:

\[
\begin{align*}
A & \subseteq B \\
A \cap A' & \subseteq B \\
A(c) & \\
A & \subseteq \bot \\
\top & \subseteq B \\
P(c, d) & \\
A & \subseteq \exists P.B \\
\exists P.\top & \subseteq B
\end{align*}
\]

where \( A, A', \) and \( B \) are class names, and \( P \) is a property or an inverse property.
Rules for Rewriting Queries

- Derive new queries by **replacing** atoms:

  \[ Q_{\exists} \quad \frac{E(x)}{D(x)} : D \subseteq E \in O \]

  \[ Q_{\bot} \quad \frac{D_1 \cap D_2(x)}{D_1(x) \quad D_2(x)} \]

  \[ Q_{\text{inv}} \quad \frac{P(x, y)}{P^-(y, x)} \]

  \[ Q_{\top} \quad \frac{\top(x)}{} \]

  \[ Q_{\exists}^+ \quad \frac{\exists P. \top(x) \quad \exists P^-. \top(y) \quad P(x, y) \quad P^-(y, x) \quad B(y)}{\exists P. B(x)} \quad \text{y a non-distinguished variable that occurs only in the query atoms in the premise; } \exists P. B \text{ occurs in } O \]

  plus any rule obtained from \( Q_{\exists}^+ \) by leaving away some (but not all) of the premises
Example Rewriting for QL

FelisCatus ⊆ ∃preysOn.Animal
SerinusCanaria ⊆ Animal
FelisCatus(silvester)
FelisCatus(tom)
SerinusCanaria(tweety)
preysOn(silvester, tweety)
Example Rewriting for QL

FelisCatus ⇐ ∃preysOn.Animal
SerinusCanaria ⇐ Animal
FelisCatus(silvester)
FelisCatus(tom)
SerinusCanaria(tweety)
preysOn(silvester, tweety)

∃y. FelisCatus(x) ∧ preysOn(x, y) ∧ Animal(y)
∃y. FelisCatus(x) ∧ preysOn(x, y) ∧ SerinusCanaria(y)
∃y. FelisCatus(x) ∧ preysOn¬(y, x) ∧ Animal(y)
∃y. FelisCatus(x) ∧ preysOn¬(y, x) ∧ SerinusCanaria(y)
  FelisCatus(x) ∧ ∃preysOn.Animal(x)
  FelisCatus(x)
Missing Bits

- Check if ontology is consistent
  - Check if the “query” \( \exists y. \bot(y) \) is entailed

- Allow query simplification by unification
  - Factorise query atoms
Correctness of the QL Rewriting Calculus

- **Soundness**: easy
- **Termination**: not hard (query building blocks finite)
- **Correctness**: not entirely trivial
  - Construct a universal model, step by step
    (may be infinite now!)
  - Every query match can be found in this model
    → can also be found in the partially constructed model after
    some number n of construction steps
  - Show that there is a rewritten query that has a match after
    only n-1 construction steps
  - Induction: some rewriting matches at n=0 (assertions in O)
The Limits of Lightweight Ontologies
Tiny OWL Profiles: Possible Extensions

- OWL RL and QL allow inverse properties, EL doesn't.
- Features for sub- and superclasses:

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Tiny OWL Profiles: Possible Extensions

- OWL RL and QL allow inverse properties, EL doesn't.
- Features for sub- and superclasses: **more is possible**

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Unions are Hard

- No tractable language can have $\Pi$ in subclasses and $\sqcup$ in superclasses

- Proof idea: Show NP-hardness by expressing 3SAT in OWL

- Try it at home (solution in Section 4.3 lecture notes)
Universal + Existential = Exponential

- Reasoning in any ontology language with
  - $\exists$ and $\forall$ in subclasses, and
  - $\forall$ and $\exists$ in superclasses
  is ExpTime-hard.

- This covers the union of RL and EL and the union of RL and QL

- How can we show this?
Alternation (1)

- An **Alternating Turing Machine (ATM)** is a non-deterministic TM whose states are partitioned into two sets of **existential states** and **universal states**.

- **Intuition:**
  - Existential state: the ATM nondeterministically picks one possible transition to move on
  - Universal state: the ATM branches into many ATMs that explore each possible transition
Alternation (2)

- A configuration of an ATM is **accepting** if:
  - it is in an existential state and one of the possible transitions leads to an accepting state, or
  - it is in a universal state and all of the possible transitions lead to an accepting state.
  
  (note: inductive definition; universal states with no transitions are accepting)

- An ATM accepts an input if the initial state is accepting on this input.
Alternation (3)

- Time and space complexity for ATMs defined as usual (considering the time/space used by a single sequence of choices, whether existential or universal)

- What makes ATMs so interesting for us:
  - ALogSpace = PTime
  - APTime = PSpace
  - APSpace = ExpTime
Simulating a PSpace ATM with OWL

- **Input:** an ATM and an input word

- **Goal:**
  - Construct an OWL ontology that derives a certain entailment iff the ATM can accept the input in polynomial space.
  - The construction should only take polynomial time
Simulating a PSpace ATM with OWL

- **Idea:** individuals represent ATM configurations, classes describe configurations, properties model transitions
- **Encoding:**
  - $A_q$: the ATM is in state $q$
  - $H_i$: the ATM head is at position $i$
  - $C_{\sigma,i}$: the tape position $i$ contains symbol $\sigma$
  - $A_{\text{cc}}$: the configuration is accepting
  - $I_w$: the initial configuration for input word $w$
  - $S_{\delta}$: property linking to configuration obtained by applying transition $\delta$
Simulating a PSpace ATM with OWL

(1) Left and right transition rules
\[ A_q \cap H_i \cap C_{\sigma, i} \sqsubseteq \exists \delta. (A_{q'} \cap H_{i+1} \cap C_{\sigma', i}) \quad \text{if } \delta = \langle q, \sigma, q', \sigma', r \rangle \text{ and } i < p(|w|) - 1 \]
\[ A_q \cap H_i \cap C_{\sigma, i} \sqsubseteq \exists \delta. (A_{q'} \cap H_{i-1} \cap C_{\sigma', i}) \quad \text{if } \delta = \langle q, \sigma, q', \sigma', l \rangle \text{ and } i > 0 \]

(2) Memory
\[ H_j \cap C_{\sigma, i} \sqsubseteq \forall \delta. C_{\sigma, i} \quad \text{if } i \neq j \]

(3) Final configurations
\[ A_q \cap H_i \cap C_{\sigma, i} \sqsubseteq \text{Acc} \quad \text{if there is no transition from } q \text{ and } \sigma \]

(4) Existential acceptance
\[ A_q \sqsubseteq \exists \delta. \text{Acc} \sqsubseteq \text{Acc} \quad \text{if } q \in E \]

(5) Universal acceptance
\[ A_q \cap H_i \cap C_{\sigma, i} \cap \bigcap_{\delta \in \Delta(q, \sigma)} \exists \delta. \text{Acc} \sqsubseteq \text{Acc} \quad \text{if } q \in U \text{ and where } \Delta(q, \sigma) \text{ is the set of all transitions from } q \text{ and } \sigma \]
Simulating a PSpace ATM with OWL

- Finally, we define the initial configuration for input $w$:  
  \( p \) defines the polynomial space bound for the ATM

\[
l_w := A_{q_0} \sqcap H_0 \sqcap C_{\sigma_{0,0}} \sqcap \ldots \sqcap C_{\sigma_{|w|-1,|w|-1}} \sqcap C_{\Box,|w|} \sqcap \ldots \sqcap C_{\Box,p(|w|)-1},
\]

- One can show (Section 4.5 of lecture notes):  
  The ontology implies $l_w \sqsubseteq Acc$ iff the ATM accepts $w$ in polynomial space.
EL + QL = ExpTime

- Only a small change is needed in the ATM simulation.

Replace:

(2) Memory

\[ H_j \cap C_{\sigma,i} \sqsubseteq \forall S_\delta . C_{\sigma,i} \quad \text{if } i \neq j \]

By:

\[ \exists S_\delta^- . (H_j \cap C_{\sigma,i}) \sqsubseteq C_{\sigma,i} \quad \text{if } i \neq j \]
Further Features of all OWL Profiles

- **Datatypes**
  - Many types (numbers, strings, dates, ...)
  - Used with DataProperties
  - Datatype expressions usable like class expressions
  - Restrictions to avoid non-determinism

- **Property Hierarchies**
Further Features of OWL EL and OWL RL

- **Property Chains**
  - Generalisation of transitivity
  - Example: hasParent o hasBrother ⊑ hasUncle
  - Subject to global restrictions in OWL DL

- **Equality**
  - State that two individuals are the same or different

- **Nominals**
  - Classes with exactly one instance, given by an individual
  - Example: ∃ livesIn.{europe} ⊑ European
Further Features of OWL EL and OWL RL

- **Functional Properties**
  - Properties that have at most one value
  - Limited to DataProperties in OWL EL
  - Missing in lecture notes

- **Keys**
  - “Rules” that imply the equality of individuals
  - Semantics restricted to named individuals
  - No description logic syntax
  - Example: HasKey( Person hasName birthday )
Further Features of OWL EL

- Local Reflexivity (Self)
  - Example: CEO $\sqsubseteq \exists$ supervisedBy.Self
  - Can be used to refer to classes in property chains:
    
    $$\text{Man} \sqsubseteq \exists \text{manProperty}.\text{Self}$$
    $$\text{manProperty o hasChild} \sqsubseteq \text{fatherOf}$$

- Not in OWL RL (no technical reason) or OWL QL (technical status not known, but should work)
Sugar

- Many OWL features can also be expressed by using other features → Syntactic sugar

- Can be more efficient for encoding something

- What is sugar depends on the available features (for example: $\Pi$ is sugar if $\sqcap$ and $\neg$ are available)
Summary & Conclusions
Summary: Reasoning in the Profiles

- Reasoning with the OWL 2 Profiles
  - Saturation (bottom-up): EL and RL
  - Rewriting (top-down): QL
    → other approaches possible in each case!

- Completeness of inference methods:
  - Relate computation to (canonical/universal) models
  - Main tool: (structural) induction

- Optimisation on several levels is important
Summary: Extending the Profiles

- Various features can be added
- Some features are generally problematic
  - Unions (in superclasses)
  - Combination of universals and existentials
  - Combination of inverses and existentials
- Hardness by simulating hard problems in OWL
  - ATMs as a powerful tool
Conclusions