Human Reasoning and Computational Logic

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- Human Reasoning
- The Suppression Task
- The New Model
- The Selection Task
- Discussion

"Logic is everywhere ..."
Human Reasoning – Two Examples

▶ Instructions on boarding card distributed at Amsterdam Schiphol Airport:

▷ If it’s thirty minutes before your flight departure, make your way to the gate. As soon as the gate number is confirmed, make your way to the gate.

▶ Notice in London Underground:

▷ If there is an emergency then you press the alarm signal bottom. The driver will stop if any part of the train is in a station.

▶ Observations

▷ Intended meaning differs from literal meaning.
▷ Rigid adherence to classical logic is no help in modeling the examples.
▷ There seems to be a reasoning process towards more plausible meanings.

► The driver will stop the train in a station if the driver is alerted to an emergency and any part of the train is in the station.

Kowalski: Computational Logic and Human Life: How to be Artificially Intelligent. Cambridge University Press 2011
The Suppression Task – Part I

Byrne: Suppressing Valid Inferences with Conditionals.
Cognition 31, 61-83: 1989

Conditionals

LE  If she has an essay to write then she will study late in the library.
LT  If she has a textbook to read then she will study late in the library.
LO  If the library stays open then she will study late in the library.

Facts

E  She has an essay to write.
¬E  She does not have an essay to write.

Will she study late in the library?  □ yes  □ no  □ I don’t know

<table>
<thead>
<tr>
<th>Conditionals</th>
<th>Facts</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE</td>
<td>E</td>
<td>96%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LT</td>
<td>E</td>
<td>96%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LO</td>
<td>E</td>
<td>38%</td>
<td></td>
</tr>
<tr>
<td>LE</td>
<td>¬E</td>
<td>46%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LT</td>
<td>¬E</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LO</td>
<td>¬E</td>
<td>63%</td>
<td></td>
</tr>
</tbody>
</table>

Classical logic is inadequate!
Reasoning Towards an Appropriate Logical Form

► Reconsider the case LE & LO.

► Context independent rules
  ▶ If she has an essay to write and the library is open
    then she will study late in the library.
  ▶ If the library is open and she has a reason for studying in the library
    then she will study late in the library.

► Context dependent rule plus exception
  ▶ If she has an essay to write then she will study late in the library.
    However, if the library is not open, then she will not study late in the library.
  ▶ The last sentence is the contrapositive of the converse of LO!
A New Computational Model

► Can we find a logic which adequately models human reasoning?

► Approach
  ▶ Reasoning towards an appropriate logical form
    ▶▶ Logic programs
  ▶ Weak completion semantics
    ▶▶ Non-monotonicity
  ▶ Three-valued Łukasiewicz logic
    ▶▶ Least models
  ▶ An appropriate semantic operator
    ▶▶ Least fixed points are least models
    ▶▶ Least fixed points can be computed by iterating the operator
  ▶ Reasoning with respect to the least models
  ▶ Abduction
Logic Programs

► Preliminaries

▷ An atom is an atomic propositions.
▷ A literal is either an atom or its negation.
▷ \( \top \) and \( \bot \) denote truth and falsehood, respectively.

► A (logic) program is a finite set of rules.

▷ A rule is an expression of the form \( A \leftarrow B_1 \land \cdots \land B_n \), where \( n \geq 1 \), \( A \) is an atom, and each \( B_i \), \( 1 \leq i \leq n \), is either a literal, \( \top \) or \( \bot \).

▷ \( A \) is called head and \( B_1 \land \cdots \land B_n \) body of the rule.

▷ Rules of the form \( A \leftarrow \top \) are called positive facts.

▷ Rules of the form \( A \leftarrow \bot \) are called negative facts.
Reasoning Towards an Appropriate Logical Form


▶ Represent conditionals as licences for implications

\[
\begin{align*}
{\text{LE \& E}} & \quad \{\ell \leftarrow e \land \neg ab_1, \; ab_1 \leftarrow \bot, \; e \leftarrow \top\} \\
{\text{LE \& LT \& E}} & \quad \{\ell \leftarrow e \land \neg ab_1, \; ab_1 \leftarrow \bot, \; \ell \leftarrow t \land \neg ab_2, \; ab_2 \leftarrow \bot, \; e \leftarrow \top\}
\end{align*}
\]

▶ Reason about additional premises

\[
\begin{align*}
{\text{LE \& LO \& E}} & \quad \{\ell \leftarrow e \land \neg ab_1, \; ab_1 \leftarrow \neg o, \; \ell \leftarrow o \land \neg ab_2, \; ab_2 \leftarrow \neg e, \; e \leftarrow \top\}
\end{align*}
\]
Completion

Let \( \mathcal{P} \) be a program. Consider the following transformation:

1. All rules with the same head \( A \leftarrow \text{Body}_1, A \leftarrow \text{Body}_2, \ldots \) are replaced by \( A \leftarrow \text{Body}_1 \lor \text{Body}_2 \lor \ldots \).
2. If an atom \( A \) is not the head of any rule in \( \mathcal{P} \) then add \( A \leftarrow \bot \).
3. All occurrences of \( \leftarrow \) are replaced by \( \leftrightarrow \).

The resulting set is called completion of \( \mathcal{P} \) or \( c \mathcal{P} \).

If step 2 is omitted then the resulting set is called weak completion of \( \mathcal{P} \) or \( wc \mathcal{P} \).

Completion versus weak completion

\[
\begin{align*}
\text{c} \{ p \leftarrow q \} &= \{ p \leftrightarrow q, q \leftrightarrow \bot \} = \text{wc} \{ p \leftarrow q, q \leftrightarrow \bot \} \\
&\neq \{ p \leftrightarrow q \} = \text{wc} \{ p \leftarrow q \}
\end{align*}
\]

A logic based on (weak) completion is non-monotonic.

\[
\begin{align*}
\text{wc} \{ p \leftarrow q, q \leftrightarrow \bot \} &= \{ p \leftrightarrow q, q \leftrightarrow \bot \} \models \neg q \\
\text{wc} \{ q \leftarrow \top, p \leftarrow q, q \leftrightarrow \bot \} &= \{ p \leftrightarrow q, q \leftrightarrow \bot \lor \top \} \models q
\end{align*}
\]
Three-Valued Interpretations

▶ A (three-valued) interpretation assigns to each formula a value from \{\top, \bot, \text{U}\}. It is represented by \langle I^\top, I^\bot \rangle, where

- \( I^\top \) contains all atoms which are mapped to \( \top \),
- \( I^\bot \) contains all atoms which are mapped to \( \bot \),
- \( I^\top \cap I^\bot = \emptyset \).
- All atoms which occur neither in \( I^\top \) nor \( I^\bot \) are mapped to \( \text{U} \).


\[
\text{U} \leftarrow_{3K} \text{U} = \text{U}
\]

▶ Łukasiewicz: O logice trójwartościowej. Ruch Filozoficzny 5, 169-171: 1920

\[
\text{U} \leftarrow_{3Ł} \text{U} = \top
\]

▶ Knowledge ordering

\[
\langle I^\top, I^\bot \rangle \preceq \langle J^\top, J^\bot \rangle \quad \text{iff} \quad I^\top \subseteq J^\top \text{ and } I^\bot \subseteq J^\bot
\]
Logic Programs under Three-Valued Łukasiewicz Semantics

Fitting: A Kripke-Kleene Semantics for Logic Programs.
Journal of Logic Programming 2, 295-312: 1985

Let $\mathcal{I}$ denote the set of all three-valued interpretations. 
$(\mathcal{I}, \preceq)$ is a complete semi-lattice.

$\langle I^\top, I^\bot \rangle \cap \langle J^\top, J^\bot \rangle = \langle I^\top \cap J^\top, I^\bot \cap J^\bot \rangle$

H., Kencana Ramli:
Logic Programs under Three-Valued Łukasiewicz’s Semantics.

The model intersection property holds for each program $\mathcal{P}$, 
i.e., $\cap \{I \mid I \models_{3\mathcal{L}} \mathcal{P} \} \models_{3\mathcal{L}} \mathcal{P}$.

The model intersection property extends to weakly completed programs.

Each weakly completed program has a least model.
Reasoning wrt the Least Model of the Weak Completion of a Program

- **LE & E**

  \[ \text{wc } \{ \ell \leftarrow e \land \neg ab_1, \ ab_1 \leftarrow \bot, \ e \leftarrow \top \} = \{ \ell \leftrightarrow e \land \neg ab_1, \ ab_1 \leftrightarrow \bot, \ e \leftrightarrow \top \} \]

  ▶ Its least model is \( \langle \{ e, \ell \}, \{ ab_1 \} \rangle \).

  ▶ It does entail \( \ell \).

- **LE & LO & E**

  \[ \text{wc } \{ \ell \leftarrow e \land \neg ab_1, \ ab_1 \leftarrow \neg o, \ e \leftarrow o \land \neg ab_2, \ ab_2 \leftarrow \neg e, \ e \leftarrow \top \} = \{ \ell \leftrightarrow (e \land \neg ab_1) \lor (o \land \neg ab_2), \ ab_1 \leftrightarrow \neg o, \ ab_2 \leftrightarrow \neg e, \ e \leftrightarrow \top \} \]

  ▶ Its least model is \( \langle \{ e \}, \{ ab_2 \} \rangle \).

  ▶ It does not entail \( \ell \).

- **Weak completion semantics (WCS) appears to be adequate!**
Computing the Least Models of Weakly Completed Programs

How can we compute the least models of weakly completed programs?

Consider the following immediate consequence operator:
\[ \Phi_{\mathcal{P}}(I) = \langle J^T, J^\perp \rangle, \]
where
\[ J^T = \{ A | \text{there exists } A \leftarrow \text{Body} \in \mathcal{P} \text{ with } I(\text{Body}) = \top \} \] and
\[ J^\perp = \{ A | \text{there exists } A \leftarrow \text{Body} \in \mathcal{P} \text{ and for all } A \leftarrow \text{Body} \in \mathcal{P} \text{ we find } I(\text{Body}) = \bot \}. \]

Note \( \Phi_{\mathcal{P}} \) ‘without the red condition’ is the Fitting operator \( \Phi_F \) (Fitting 1985).

Theorem (H., Kencana Ramli 2009)
(1) \( \Phi_{\mathcal{P}} \) is monotone on \( (\mathcal{I}, \preceq) \).
(2) \( \Phi_{\mathcal{P}} \) is continuous
and, hence, admits a least fixed point denoted by \( \text{lfp } \Phi_{\mathcal{P}} \).
(3) \( \text{lfp } \Phi_{\mathcal{P}} \) can be computed by iterating \( \Phi_{\mathcal{P}} \) on \( \langle \emptyset, \emptyset \rangle \).
(4) \( \text{lm}_{3\mathcal{L}} \text{ wc } \mathcal{P} = \text{lfp } \Phi_{\mathcal{P}} \).
The Suppression Task – Part II

Byrne: Suppressing Valid Inferences with Conditionals. Cognition 31, 61-83: 1989

Conditionals
LE  If she has an essay to write then she will study late in the library.
LT  If she has a textbook to read then she will study late in the library.
LO  If the library stays open then she will study late in the library.

Facts
L   She will study late in the library.
¬L  She will not study late in the library.

Has she an essay to write?  □ yes  □ no  □ I don’t know

<table>
<thead>
<tr>
<th>Conditionals</th>
<th>Facts</th>
<th>Yes</th>
<th>No</th>
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<tbody>
<tr>
<td>LE</td>
<td>L</td>
<td>53%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LT</td>
<td>L</td>
<td>16%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LO</td>
<td>L</td>
<td>55%</td>
<td></td>
</tr>
<tr>
<td>LE</td>
<td>¬L</td>
<td>69%</td>
<td></td>
</tr>
<tr>
<td>LE &amp; LT</td>
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<td></td>
</tr>
<tr>
<td>LE &amp; LO</td>
<td>¬L</td>
<td>44%</td>
<td></td>
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</table>
Abduction

▶ LE & LT & L

- Knowledge base \( \{ \ell \leftarrow e \land \neg ab_1, \ ab_1 \leftarrow \bot, \ \ell \leftarrow t \land \neg ab_2, \ ab_2 \leftarrow \bot \} \)
- Observation \( \ell \)
- Abducibles \( \{ e \leftarrow \top, \ e \leftarrow \bot, \ t \leftarrow \top, \ t \leftarrow \bot \} \)
- Two minimal explanations \( \{ e \leftarrow \top \} \) and \( \{ t \leftarrow \top \} \)
- Reasoning credulously we conclude \( e \).
- Reasoning sceptically we cannot conclude \( e \).
- Byrne 1989 only 16% conclude \( e \).

▶ Sceptical reasoning appears to be adequate!


Weak Completion versus Well-Founded Semantics (1)

- Dietz, H., Wernhard: Modeling the Suppression Task under Weak Completion and Well-Founded Semantics: Journal of Applied Non-Classical Logics (to appear)

- A program is **tight** if it does not contain positive cycles.
- All programs for the suppression (and the selection) task are **tight**.
- **Theorem** Let \( \mathcal{P} \) be a tight program and \( I \) an interpretation. The following statements are equivalent:
  - \( I \) is a least model of the weak completion of \( \mathcal{P} \).
  - \( I \) is a well-founded model of \( \mathcal{P}' \), where \( \mathcal{P}' \) is obtained from \( \mathcal{P} \) by deleting all negative facts and adding for each undefined predicate symbol \( A \) occurring in \( \mathcal{P} \) the rules \( A \leftarrow \neg A' \) and \( A' \leftarrow \neg A \), where \( A' \) is a new symbol.

- Well-founded semantics (WFS) appears to be adequate if conditionals do not contain positive cycles!
Weak Completion versus Well-Founded Semantics (2)

How do humans reason with positive cycles?

- If they open the window, then they open the window.
- If they open the window, then it is cold.
  If it is cold, then they wear their jackets.
  If they wear their jackets, then they open the windows.

Psychological study

- We presented conditionals with positive cycles of length one, two and three, asked whether embedded propositions or their negations are entailed.

Preliminary results

<table>
<thead>
<tr>
<th>length</th>
<th>positive (WFS)</th>
<th>negative (WFS)</th>
<th>unknown (WCS)</th>
<th>response time</th>
</tr>
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<td>1</td>
<td>75 %</td>
<td>0 %</td>
<td>25 %</td>
<td>5257 msec</td>
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<tr>
<td>2</td>
<td>60 %</td>
<td>3 %</td>
<td>37 %</td>
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<tr>
<td>3</td>
<td>55 %</td>
<td>4 %</td>
<td>41 %</td>
<td>11680 msec</td>
</tr>
</tbody>
</table>

Humans consider positive cycles of length one as facts.

The longer the cycles, the more likely is the answer ’unknown’.

Almost nobody entailed negative propositions.
The Selection Task – Abstract Case

- Wason: Reasoning about a Rule.  

- Consider cards which have a letter on one side and a number on the other side.

```
D  F  3  7
```

- Consider the rule:

  *if there is a D on one side, then there is a 3 on the other side.*

- Which cards do you have to turn in order to show that the rule holds?

  - Only 10% of the subjects give the logically correct solutions.
An Analysis

- Almost everyone (89%) correctly selects D.
  - Corresponds to modus ponens in classical logic.

- Almost everyone (84%) correctly does not select F.
  - *Because the condition does not mention F.*

- Many (62%) incorrectly select 3.
  - "If there is a 3 on one side, then there is a D on the other side." 
  - Converse of the given conditional.

- Only a small percentage of subjects (25%) correctly selects 7.
  - "If the number on one side is not 3, then the letter on the other side is not D." 
  - Contrapositive of the given conditional.
The Selection Task – Social Case


Consider cards which have a person’s age on the one side and a drink on the other side.

beer  coke  22yrs  16yrs

Consider the rule:

*If a person is drinking beer, then the person must be over 19 years of age.*

Which cards do you have to turn in order to show that the rule holds?

Most people solve this variant correctly.
A Computational Logic Approach to the Selection Task

The computational logic approach to model human reasoning can be extended to adequately handle the selection task

▷ if the social case is understood as a social constraint and
▷ if the abstract case is understood as a belief.

Kowalski: Computational Logic and Human Life: How to be Artificially Intelligent. Cambridge University Press: 2011

Discussion

► Logic appears to be adequate for human reasoning if
  ▶ weak completion semantics,
  ▶ Łukasiewicz logic,
  ▶ the Stenning and van Lambalgen semantic operator, and
  ▶ sceptical abduction are used.

► Human reasoning is modeled by
  ▶ reasoning towards an appropriate logic program and, thereafter,
  ▶ reasoning with respect to the least model of its weak completion.

► This approach matches data from studies in human reasoning.

► There is a connectionist encoding.

► There are many interesting and challenging open questions.
Some Open Problems (1)

- **Negation**
  - How is negation treated in human reasoning?

- **Errors**
  - How can frequently made errors be explained in the proposed approach?

- **Łukasiewicz logic**
  - Is the Łukasiewicz logic adequate?

- **Completion**
  - Under which conditions is human reasoning adequately modeled by completion and/or weak completion?
Some Open Problems (2)

- **Contractions**
  - Do humans exhibit a behavior which can be adequately modeled by contractional semantic operators?

- **Explanations**
  - Do humans consider minimal explanations?
  - In which order are (minimal) explanations generated by humans if there are several?
  - Does attention play a role in the selection of (minimal) explanations?

- **Reasoning**
  - Do humans reason sceptically or credulously?
  - How does a connectionist realization of sceptical reasoning looks like?